UV/IR Mixing and Black Hole Thermodynamics

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Abstract

The goal of this paper is to investigate the final stage of black hole evaporation process in the framework of Lorentz violating Modified Dispersion Relations(MDRs). As a consequence of MDRs, the high energy sector of the underlying field theory does not decouple from the low energy sector, the phenomenon which is known as UV/IR mixing. In the absence of exact supersymmetry, we derive a modified dispersion relation which shows UV/IR mixing by a novel energy dependence. Then we investigate the effects of these type of MDRs on the thermodynamics of a radiating noncommutative Schwarzschild black hole. The final stage of black hole evaporation obtained in this framework is compared with existing pictures.

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1 Introduction

There are some evidences from alternative approaches to quantum gravity which indicate that Lorentz symmetry is not an exact symmetry of the nature[1-5]. One possible framework to incorporate this Lorentz invariance violation in the equations of physics is the modification of the standard dispersion relation[2,4,6-8]. The resulting modified dispersion relations(MDRs) show some interesting aspects of Planck scale physics such as UV/IR mixing. Based on this idea, the high energy sector of the theory does not decouple from the low energy sector. These features may reflect the fact that spacetime at quantum gravity level has a granular structure. This granular feature can describe the energy dependence of correction terms to standard dispersion relation[7]. The effects of MDRs on various aspects of quantum gravity problem have been studied extensively[9-14]. From a phenomenological point of view, these MDRs can be considered the basis of some important test theories which can justify alternative approaches to quantum gravity problem[15,16]. One of the important outcome of MDRs is the possible interpretation of the astrophysical anomalies such as GZK [1] and TeV photon anomaly[4,19]. Since MDRs are common feature of all promising quantum gravity candidates, it would be interesting to examine the effects of them on a key problem of quantum gravity, that is, black hole thermodynamics. Based on this view point, some authors have applied MDRs to the formulation of black hole physics[9,10]. An elegant application of MDRs to black hole physics and some related issues is provided by Amelino-Camelia *et al*[9]. They have used a formulation of MDRs which is common in existing literature and has the following form

$$(\vec{p})^2 \simeq E^2 - \mu^2 + \alpha_1 L_P E^3 + \alpha_2 L_P^2 E^4 + O(L_P^3 E^5) \tag{1}$$

where μ is related to the rest mass and α_i are quantum gravity model-dependent constants that may take different values for different particles[7]. These type of MDRs have several implications on the final stage of black hole evaporation. Comparison between the results of MDRs for black hole thermodynamics and the standard result of string theory, imposes some important constraints on the functional form of MDRs as given in (1)[9,20]. For instance, as has been shown in [10], only even powers of energy should be present in relations such as (1).

Based on MDRs[9,10,20], the generalized uncertainty principle[21-24], and also noncommutative geometry[25-27], the current view point on the final stage of a black hole

[1] The Greisen-Zatsepin-Kuzmin limit (GZK limit) is a theoretical upper limit on the energy of cosmic rays from distant sources[4,5,6,16,17,18].

evaporation can be summarized as follows:
Black hole evaporates by emission of Hawking radiation in such a way that in the final stage of evaporation, it reaches to a maximum temperature before cooling down and finally reaches to a stable remnant with zero entropy.
The purpose of this paper is to re-examine black hole thermodynamics within a combination of MDRs and space noncommutativity. Using a general formulation of modified dispersion relations in the language of noncommutative geometry, we obtain a MDR which contains a novel energy dependence relative to relation (1). This type of MDR consists of modification terms which are functions of inverse of powers of energy and show an explicit UV/IR mixing. We apply our MDR to the issue of black hole thermodynamics and compare our results with existing picture.
The paper is organized as follows: In section 2 we use noncommutative space framework to obtain a new MDR with a novel energy dependence. Section 3 applies our MDR to the issue of black hole thermodynamics. The paper follows by discussion and results in section 4.

2 Modified Dispersion Relations and UV/IR Mixing

In this section, using the notion of space noncommutativity, we find a new modified dispersion relation indicating explicit UV/IR mixing. A noncommutative space can be defined by the coordinate operators satisfying the following commutation relation[28-32]

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} + i\rho^{\beta}_{\mu\nu}x_\beta \tag{2}$$

In the spacial case where $\rho^{\beta}_{\mu\nu}$ vanishes, we find the canonical noncommutative spacetime with the following algebraic structure

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} \tag{3}$$

where $\hat{x}$'s are the coordinate operators and $\theta_{\mu\nu}$ is an antisymmetric matrix whose elements have dimension of $(length)^2$.
Modification of standard field theory due to spacetime noncommutativity has been studied extensively(see for example [30,32,33]). One important consequence of noncommutative field theory is emergence of the so called UV/IR mixing; the high energy sector of the theory does not decouple from the low energy sector. This UV/IR mixing can be addressed

via modification of standard dispersion relations. Within the canonical noncommutative field theory, this modified dispersion relation attains the following form[7]

$$m^2 \simeq E^2 - \vec{p}^2 + \frac{\alpha_1}{p^\mu \theta_{\mu\nu} \theta^{\nu\sigma} p_\sigma} + \alpha_2 m^2 \ln(p^\mu \theta_{\mu\nu} \theta^{\nu\sigma} p_\sigma) + ..., \tag{4}$$

where $\theta_{ij} = \frac{1}{2}\epsilon_{ijk}\theta^k$. The α_i are parameters dependent on various aspects of the field theory and can take different values for different particles since dispersion relation is not universal. Using the identity

$$\epsilon_{ijr}\epsilon_{iks} = \delta_{jk}\delta_{rs} - \delta_{js}\delta_{rk}, \tag{5}$$

one finds

$$p^\mu \theta_{\mu\nu} \theta^{\nu\sigma} p_\sigma = -\frac{1}{4}\Big(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\Big). \tag{6}$$

Therefore, equation (4) can be written as follows

$$E^2 \simeq m^2 + \vec{p}^2 + \frac{4\alpha_1}{\Big(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\Big)} - \alpha_2 m^2 \ln\Big[-\frac{1}{4}\Big(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\Big)\Big]. \tag{7}$$

This MDR can be singular in the infrared regime as a result of UV/IR mixing. In the case of exact supersymmetry, part of this infrared singularity can be removed by setting $\alpha_1 = 0$. The case with $\alpha_1 \neq 0$ has not been considered in literature but as we will show it has some novel implication in the spirit of black hole thermodynamics. So, in which follows we consider the non-supersymmetric case where $\alpha_1 \neq 0$. On the other hand, except for situation where $\vec{p}.\vec{\theta} = p\theta$, the quantity $-\frac{1}{4}\Big(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\Big)$ is negative, therefore the last term is imaginary. As an example of this situation, note that phonons in a fluid flow can propagate with an MDR which shows imaginary terms if viscosity is taken into account. In this paper we don't consider these extreme situations therefore in which follows, we consider the case where $\alpha_2 = 0$. The parameter m is directly related to the rest energy, and in the high energy regime we can neglect it. Considering these points, we find

$$E^2 \simeq \vec{p}^2 + \frac{4\alpha_1}{\Big(p^2\theta^2 - (\vec{p}.\vec{\theta})^2\Big)} \tag{8}$$

where $p^2 = \vec{p}.\vec{p}$ and $\theta^2 = \vec{\theta}.\vec{\theta}$. If we set $\theta_3 = \theta$ and assuming that remaining components of θ all vanish (which can be done by a rotation or a re-definition of the coordinates), then $\vec{p}.\vec{\theta} = p_z\theta$. In this situation equation (8) can be written as follows

$$E^2 \simeq (p_x^2 + p_y^2 + p_z^2) + \frac{4\alpha_1}{(p_x^2 + p_y^2)\theta^2}. \tag{9}$$

Assuming an isotropic case where $p_x = p_y = p_z = \tilde{p}$, we find

$$3\tilde{p}^2 + \frac{2\alpha_1}{\tilde{p}^2\theta^2} - E^2 = 0. \tag{10}$$

This equation has two solutions for $\tilde{p}^2$. Only one of these solutions is acceptable since in standard limit where $\alpha_1 = 0$ we should recover $3\tilde{p}^2 = E^2$ (note that we have omitted the rest mass from our calculations). This solution is

$$\tilde{p}^2 = \frac{1}{6}\Big(E^2 + \sqrt{E^4 - \frac{24\alpha_1}{\theta^2}}\Big) \tag{11}$$

or

$$p^2 = \frac{1}{2}\Big(E^2 + \sqrt{E^4 - \frac{24\alpha_1}{\theta^2}}\Big). \tag{12}$$

We expand this relation up to second order of α_1 to find

$$p^2 = E^2 - \frac{6\alpha_1}{\theta^2 E^2} - \frac{36\alpha_1^2}{\theta^4 E^6} + \mathcal{O}(\frac{\alpha_1^3}{\theta^6 E^{10}}) \tag{13}$$

Our forthcoming arguments are based on this result. It provides an energy dependence which has not been pointed out in existing literature explicitly. Note that based on Heisenberg uncertainty principle, since always $E \geq \frac{1}{\delta x}$ where δx is particle position uncertainty, relation (13) is well defined. In the existing literature, following analysis of Amelino-Camelia *et al* [9], in most cases, the authors have led to consider a dispersion relation of the type

$$\vec{p}^2 \simeq E^2 + \beta_1 L_p E^3 + \beta_2 L_p^2 E^4 + \mathcal{O}(L_p^3 E^5) \tag{14}$$

where the coefficients β_i can take different values in different quantum gravity proposals. In this type of MDRs there is no dependence to the inverse powers of E. However, our result given by (13) contains modification terms with powers of inverse of E. Note also that our MDR contains even powers of energy which is in agreement with our previous finding[10]. We proceed in the line of Amelino-Camelia *et al* approach[9] but instead of their MDR as given by (14), we use our MDR (13) which mainly considers the IR limit of the noncommutative field theory. In this manner, some novel results are obtained which illuminate further the final stage of black hole evaporation. Due to different energy dependence of our MDR, we expect that thermodynamics of black hole obtained within this framework differs from black hole thermodynamics obtained in other MDRs framework. For instance, we will observe that in this framework there is no logarithmic correction

to Bekenstein-Hawking entropy-area relation. This difference may reflect some aspects of UV/IR mixing and related quantum gravity phenomena.

The relation between dp and dE obtained from (13) is as follows

$$dp = \Big[1 + a_1\Big(\frac{\alpha_1}{\theta^2 E^4}\Big) + a_2\Big(\frac{\alpha_1}{\theta^2 E^4}\Big)^2 + a_3\Big(\frac{\alpha_1}{\theta^2 E^4}\Big)^3\Big]dE \tag{15}$$

where the coefficients a_i are constant and we consider terms only up to third order of α_1. According to Heisenberg's uncertainty principle, in order to measure the particle position with precision δx one should use a photon with momentum uncertainty $\delta p \geq \frac{1}{\delta x}$. Within our framework and considering this point, one is led to the following relation

$$E \geq \frac{1}{\delta x}\Bigg[1 - a_1\Big(\frac{\alpha_1(\delta x)^4}{\theta^2}\Big) + (a_1^2 - a_2)\Big(\frac{\alpha_1(\delta x)^4}{\theta^2}\Big)^2 + (2a_1a_2 - a_3 - a_1^3)\Big(\frac{\alpha_1(\delta x)^4}{\theta^2}\Big)^3\Bigg]. \tag{16}$$

For the standard case where $a_i = 0$, this equation simplifies to $E \geq \frac{1}{\delta x}$. In which follows, we show that the modification of dispersion relation of the type (13), can lead to corrections of standard relations for entropy and temperature of the black hole, i. e. standard Bekenstein-Hawking entropy-area relations $S = \frac{A}{4}$ and $T = \frac{1}{8\pi M}$ will be modified as a result of UV/IR mixing.

3 Black Hole Thermodynamics with Modified Dispersion Relation

The Bekenstein argument suggests that the entropy of a black hole should be proportional to its area of event horizon. The minimum increase of area when the black hole absorbs a classical particle of energy E and size s is $\Delta A \geq 8\pi E s$ [9]. When black hole absorbs a quantum particle of size s, uncertainty in position of the particle will be δx where $s \sim \delta x$. Considering a calibration factor as $\frac{\ln 2}{2\pi}$, we find

$$\Delta A \geq 4(\ln 2)E\delta x; \tag{17}$$

In the standard case $E \sim \frac{1}{\delta x}$, which leads to

$$\Delta A \geq 4(\ln 2). \tag{18}$$

Using the fact that the minimum increase of entropy is $\ln 2$ (one bit of information) and it is independent of the area A, one find

$$\frac{dS}{dA} \simeq \frac{(\Delta S)_{min}}{(\Delta A)_{min}} \simeq \frac{1}{4}. \tag{19}$$

Integration leads to standard Bekenstein result

$$S \simeq \frac{A}{4}. \tag{20}$$

To calculate entropy in the presence of UV/IR mixing, we use relations (15) and (16) to find

$$\Delta A \geq 4(\ln 2)\Bigg[1-a_1\Big(\frac{\alpha_1(\delta x)^4}{\theta^2}\Big)+(a_1^2-a_2)\Big(\frac{\alpha_1(\delta x)^4}{\theta^2}\Big)^2+(2a_1a_2-a_3-a_1^3)\Big(\frac{\alpha_1(\delta x)^4}{\theta^2}\Big)^3\Bigg]. \tag{21}$$

Now to calculate entropy and temperature of black hole we need the radius of event horizon r_H. There are two relatively different approaches to find noncommutative radius of the event horizon[25,27]. These two approaches are based on two different view points to make general relativity noncommutative(see [25] and [34] for further details). Here we use Nicollini *et al* approach to find noncommutative radius of event horizon[25]. It has been shown that noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime. As Nicolini *et al* have shown, the effect of smearing is mathematically implemented as a substitution rule: position Dirac-delta function is replaced everywhere with a Gaussian distribution of minimal width $\sqrt{\theta}$. In this framework, they have chosen the mass density of a static, spherically symmetric, smeared, particle-like gravitational source as follows

$$\rho_\theta(r) = \frac{M}{(2\pi\theta)^{\frac{3}{2}}}\exp(-\frac{r^2}{4\theta}) \tag{22}$$

As they have indicated, the particle mass M, instead of being perfectly localized at a point, is diffused throughout a region of linear size $\sqrt{\theta}$. This is due to the intrinsic uncertainty as has been shown in the coordinate commutators (3). This kind of matter source results the following static, spherically symmetric, asymptotically Schwarzschild solution of the Einstein equations[25,26]

$$ds^2 = \Bigg(1-\frac{2M}{r\sqrt{\pi}}\gamma\Big(\frac{1}{2},\frac{r^2}{4\theta}\Big)\Bigg)dt^2 - \Bigg(1-\frac{2M}{r\sqrt{\pi}}\gamma\Big(\frac{1}{2},\frac{r^2}{4\theta}\Big)\Bigg)^{-1}dr^2 - r^2(d\vartheta^2 + sin^2\vartheta d\phi^2) \tag{23}$$

where $\gamma\Big(\frac{1}{2},\frac{r^2}{4\theta}\Big)$ is the lower incomplete Gamma function:

$$\gamma\Big(\frac{1}{2},\frac{r^2}{4\theta}\Big) \equiv \int_0^{\frac{r^2}{4\theta}} t^{\frac{1}{2}}e^{-t}dt \tag{24}$$

The event horizon of this metric can be found where $g_{00}(r_H) = 0$,

$$r_H = \frac{2M}{\sqrt{\pi}}\gamma\Big(\frac{1}{2},\frac{r_H^2}{4\theta}\Big) \tag{25}$$

As it is obvious from this equation, the effect of noncommutativity in the large radius regime can be neglected, while at short distance regime one expects significant changes due to the spacetime fuzziness.

After determination of noncommutative radius of black hole event horizon, we return to our original argument on black hole thermodynamics. When a particle falls into the black hole event horizon, the particle position uncertainty will be $\delta x \sim r_H$, where r_H is the Schwarzchild radius in noncommutative spacetime. Therefore we have from (21)

$$\Delta A \geq 4(\ln 2)\Big[1 - a_1\Big(\frac{\alpha_1}{\theta^2}\Big)r_H^4 + (a_1^2 - a_2)\Big(\frac{\alpha_1}{\theta^2}\Big)^2 r_H^8 + (2a_1a_2 - a_3 - a_1^3)\Big(\frac{\alpha_1}{\theta^2}\Big)^3 r_H^{12}\Big] \qquad (26)$$

defining $A = 4\pi r_H^2$, equation (26) takes the following form

$$\Delta A \geq 4(\ln 2)\Big[1 - a_1\Big(\frac{\alpha_1}{\theta^2}\Big)\Big(\frac{A\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{4\pi^2}\Big)^2 + (a_1^2 - a_2)\Big(\frac{\alpha_1}{\theta^2}\Big)^2\Big(\frac{A\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{4\pi^2}\Big)^4$$
$$+(2a_1a_2 - a_3 - a_1^3)\Big(\frac{\alpha_1}{\theta^2}\Big)^3\Big(\frac{A\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{4\pi^2}\Big)^6\Big]. \qquad (27)$$

Now the entropy of black hole can be calculated as follows

$$\frac{dS}{dA} \simeq \frac{(\Delta S)_{min}}{(\Delta A)_{min}}$$
$$\simeq \frac{1}{4}\Big[1 + a_1\Big(\frac{\alpha_1}{\theta^2}\Big)\Big(\frac{A\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{4\pi^2}\Big)^2 + a_2\Big(\frac{\alpha_1}{\theta^2}\Big)^2\Big(\frac{A\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{4\pi^2}\Big)^4 + a_3\Big(\frac{\alpha_1}{\theta^2}\Big)^3\Big(\frac{A\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{4\pi^2}\Big)^6\Big]. \quad (28)$$

In terms of event horizon radius, this relation can be written as

$$\frac{dS}{dr_H} \simeq \Big[2\pi r_H + \frac{2a_1}{\pi}\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\theta}\Big)^2 r_H^5 + \frac{2a_2}{\pi^3}\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\theta}\Big)^4 r_H^9 + \frac{2a_3}{\pi^5}\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\theta}\Big)^6 r_H^{13}\Big], \qquad (29)$$

which integration gives

$$S \simeq \frac{A}{4} + \int\Big[\frac{2a_1}{\pi}\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\theta}\Big)^2 r_H^5 + \frac{2a_2}{\pi^3}\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\theta}\Big)^4 r_H^9$$
$$+\frac{2a_3}{\pi^5}\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\theta}\Big)^6 r_H^{13}\Big]dr_H. \qquad (30)$$

The first term in the right hand side is the standard Bekenstein entropy. But, integration of second term has no closed form and can be calculated numerically. This result

shows that black hole entropy in the presence of MDR(UV/IR mixing) has a complicated form and this form is different from the entropy-area relation obtained in other quantum gravity-based approaches(see for example results of Amelino-Camelia *et al* in [9]). Figure 1 shows the numerical calculation of entropy-event horizon relation for an evaporating black hole in Bekenstein-Hawking and the noncommutative geometry view points[2]. As this figure shows, within noncommutative geometry approach, black hole in its final stage of evaporation reaches to a zero entropy remnant.

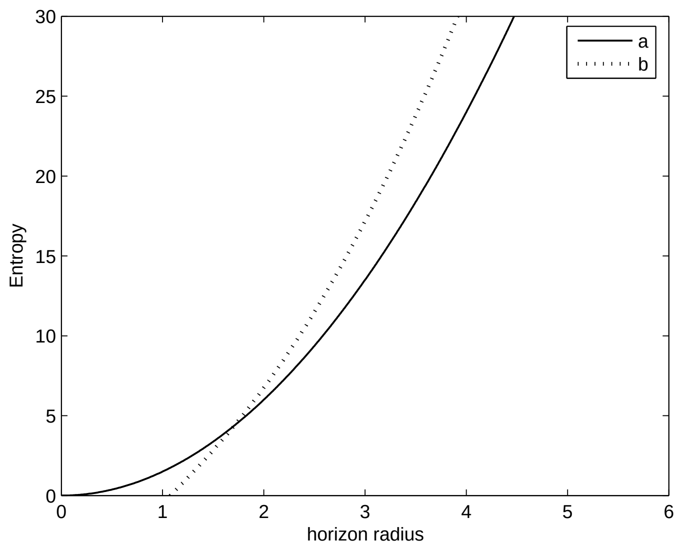

Figure 1: Black hole entropy versus its radius of event horizon: a)standard Bekenstein-Hawking result, and b)noncommutative space result. We have set $\theta = 1$ and $\alpha_1 = 1$. All a_i coefficients are assumed to be positive.

This result is in agreement with existing literature. However in our case we reach a remnant with larger mass relative to existing results such as result of Nicolini *et al*[25].

[2]As we have mentioned in page 4, α_i coefficients are parameters dependent on various aspects of the field theory and can take different values for different particles since dispersion relation is not universal. To have a qualitative behavior of our thermodynamical quantities, we have set $\alpha_1 = 1$ in figures. On the other hand, since we have set $\hbar = 1$, our noncommutative commutation relations are given by (3). With $\hbar \neq 1$ we will find $[x_\mu, x_\nu] = i\hbar\theta_{\mu\nu}$ where in this case $\theta_{\mu\nu}$ will have dimension of $\frac{(length)^2}{\hbar}$. In this regard, we have set $\theta = 1$ in our numerical calculations. This is a common approach to find qualitative description of black hole thermodynamical quantities. Currently no concrete values of these quantities are in hand since these parameters are quantum gravity-model dependent.

To proceed further, we discuss two extreme limits: commutative limit and highly non-commutative limit.

- In the large radius regime $\frac{r_H^2}{4\theta} \gg 1$ (commutative regime), the γ function can be solved to the first order of approximation to find

$$\gamma(\frac{1}{2}, \frac{r_H^2}{4\theta}) = -\frac{2\sqrt{\theta}}{r_H} e^{-\frac{r_H^2}{4\theta}}. \tag{31}$$

  Therefore from equation (29) we find

$$\frac{dS}{dr_H} \simeq 2\pi r_H \Big[1 + \frac{a_1}{\pi^2}\Big(16\alpha_1 e^{-\frac{r_H^2}{\theta}}\Big) + \frac{a_2}{\pi^4}\Big(16\alpha_1 e^{-\frac{r_H^2}{\theta}}\Big)^2 + \frac{a_3}{\pi^6}\Big(16\alpha_1 e^{-\frac{r_H^2}{\theta}}\Big)^3\Big]. \tag{32}$$

  In this case entropy can be calculated as follows

$$S \simeq \frac{A}{4} - \frac{a_1\theta}{\pi}\Big(16\alpha_1 e^{-\frac{A}{4\pi\theta}}\Big) - \frac{a_2\theta}{2\pi^3}\Big(16\alpha_1 e^{-\frac{A}{4\pi\theta}}\Big)^2 - \frac{a_3\theta}{3\pi^5}\Big(16\alpha_1 e^{-\frac{A}{4\pi\theta}}\Big)^3. \tag{33}$$

  We see from this relation that for large radii with respect to $\sqrt{\theta}$, the effect of non-commutativity of spacetime is exponentially small.

- In the opposite limit where $r_H \simeq \sqrt{\theta}$, the structure of spacetime is fuzzy and the γ function in this limit (noncommutative limit) can be solved as

$$\gamma(\frac{1}{2}, \frac{r_H^2}{4\theta}) = \frac{r_H}{\sqrt{\theta}} e^{-\frac{r_H^2}{4\theta}}, \tag{34}$$

  In this case equation (29) leads to the following expression

$$\frac{dS}{dr_H} \simeq 2\pi r_H \Big[1 + \frac{a_1}{\pi^2}\Big(\frac{\alpha_1 r_H^8 e^{-\frac{r_H^2}{\theta}}}{\theta^4}\Big) + \frac{a_2}{\pi^4}\Big(\frac{\alpha_1 r_H^8 e^{-\frac{r_H^2}{\theta}}}{\theta^4}\Big)^2 + \frac{a_3}{\pi^6}\Big(\frac{\alpha_1 r_H^8 e^{-\frac{r_H^2}{\theta}}}{\theta^4}\Big)^3\Big], \tag{35}$$

  and integration leads to the following result for black hole entropy in strong non-commutative limit

$$S \simeq \frac{A}{4} - \frac{a_1\alpha_1}{\pi}\Big[24\theta + 24\Big(\frac{A}{4\pi}\Big) + \frac{12}{\theta}\Big(\frac{A}{4\pi}\Big)^2 + \frac{4}{\theta^2}\Big(\frac{A}{4\pi}\Big)^3 + \frac{1}{\theta^3}\Big(\frac{A}{4\pi}\Big)^4\Big] e^{-\frac{A}{4\pi\theta}}$$

$$-\frac{\mu a_2 \alpha_1^2}{\pi^3}\Big[\frac{3}{2}\theta + 3\Big(\frac{A}{4\pi}\Big) + \frac{3}{\theta}\Big(\frac{A}{4\pi}\Big)^2 + \frac{2}{\theta^2}\Big(\frac{A}{4\pi}\Big)^3 + \frac{1}{\theta^3}\Big(\frac{A}{4\pi}\Big)^4\Big] e^{-\frac{2A}{4\pi\theta}}$$

$$-\frac{\nu a_3\alpha_1^3}{\pi^5}\Bigg[\frac{8}{27}\theta+\frac{8}{9}\Big(\frac{A}{4\pi}\Big)+\frac{4}{3\theta}\Big(\frac{A}{4\pi}\Big)^2+\frac{4}{3\theta^2}\Big(\frac{A}{4\pi}\Big)^3+\frac{1}{2\theta^3}\Big(\frac{A}{4\pi}\Big)^4\Bigg]e^{-\frac{3A}{4\pi\theta}}, \qquad (36)$$

where we have considered only terms up to fourth order of A in brackets and μ and ν are constant. This is an interesting result which shows the modified entropy of black hole in noncommutative geometry. Two main characteristic feature of this relation are: it has Bekenstein-Hawking result as commutative limit but it contains no logarithmic correction term. It is commonly believed that entropy should have a series expansion as follows(see [10] and references therein)

$$S\simeq\frac{A}{4}-\frac{\pi\zeta}{2}\ln\frac{A}{4}+\sum_{n=1}^{\infty}c_n\Big(\frac{4}{A}\Big)^n+\mathcal{C}. \qquad (37)$$

Our analysis shows that in the presence of UV/IR mixing there are some deviation from this standard result since there is no logarithmic correction term and also there are power of event horizon area instead of inverse of power of event horizon area. The presence of logarithmic correction term which is a matter of debate in the literatures now finds a natural solution in MDRs framework.

Using the first law of black hole thermodynamics $dS=\frac{dM}{T}$, we can calculate the corrected temperature in the framework of our analysis. Considering $\frac{dS}{dM}=\frac{1}{T}$ and $r_H=2M$, the corrected temperature in noncommutative spacetime is obtained as follows

$$T\simeq\frac{1}{8\pi M}\Bigg[1-a_1\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\pi\theta}\Big)^2(4M^2)^2+(a_1^2-a_2)\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\pi\theta}\Big)^4(4M^2)^4$$
$$+(2a_1a_2-a_1^3-a_3)\Big(\frac{\sqrt{\alpha_1}\gamma^2(\frac{1}{2},\frac{r_H^2}{4\theta})}{\pi\theta}\Big)^6(4M^2)^6\Bigg]. \qquad (38)$$

For semiclassical limit $\frac{r_H^2}{4\theta}\gg 1$, the temperature using equation (31) takes the following form

$$T_H\simeq\frac{1}{4\pi r_H}\Bigg[1-a_1\Big(\frac{16\alpha_1}{\pi^2}e^{-\frac{r_H^2}{\theta}}\Big)+(a_1^2-a_2)\Big(\frac{16\alpha_1}{\pi^2}e^{-\frac{r_H^2}{\theta}}\Big)^2+(2a_1a_2+2a_1^2-a_3)\Big(\frac{16\alpha_1}{\pi^2}e^{-\frac{r_H^2}{\theta}}\Big)^3\Bigg], \qquad (39)$$

which leads to the standard relation for Hawking temperature

$$T_H=\frac{1}{4\pi r_H}. \qquad (40)$$

In noncommutative limit where $r_H \simeq \sqrt{\theta}$, using equation (34) the temperature can be written as follows

$$T_H \simeq \frac{1}{4\pi r_H}\Bigg[1-a_1\Big(\frac{\alpha_1 r_H^8 e^{-\frac{r_H^2}{\theta}}}{\pi^2\theta^4}\Big)+(a_1^2-a_2)\Big(\frac{\alpha_1 r_H^8 e^{-\frac{r_H^2}{\theta}}}{\pi^2\theta^4}\Big)^2+(2a_1a_2-a_1^3-a_3)\Big(\frac{\alpha_1 r_H^8 e^{-\frac{r_H^2}{\theta}}}{\pi^2\theta^4}\Big)^3\Bigg], \tag{41}$$

which has very different form from existing picture but as we will see its general behavior with respect to event horizon radius is the same as existing picture. In the commutative limit, when the horizon radius decreases, the temperature increases and diverges. In the opposite limit, when we consider fuzzy spacetime, the black hole before cooling down to absolute zero, reaches to a finite maximum temperature. Figure 2 shows the temperature-event horizon relation for an evaporating black hole in Bekenstein-Hawking and the noncommutative geometry view points. This result is in agreement with the results of Nicolini *et al* which have computed black hole temperature in a different view point[25].

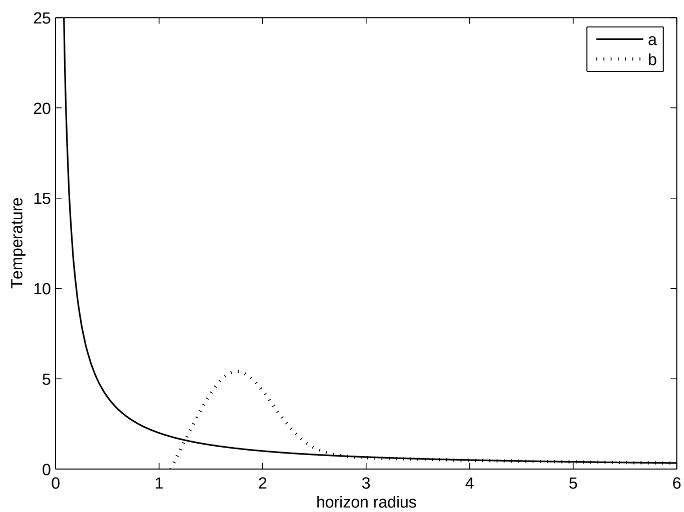

Figure 2: Black hole temperature versus its radius of event horizon in two different regimes: a)standard Bekenstein-Hawking result, and b)noncommutative space result. We have set $\theta = 1$ and $\alpha_1 = 1$. All a_i coefficients are assumed to be negative.

One point should be stressed here: for some values of constants in relation (41), it is possible to have negative temperature for final state of black hole evaporation(see figure 3). It is a well-known issue in condensed matter physics that under certain conditions, a closed system can be described by a negative temperature, and, surprisingly, be hotter than the

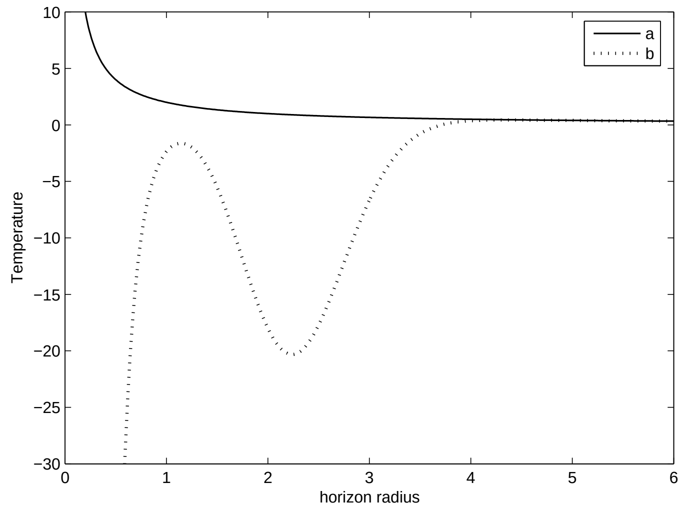

Figure 3: Black hole temperature versus its radius of event horizon in two different regimes and for the case of possible negative temperature: a)standard Bekenstein-Hawking result, and b)noncommutative space result. We have set $\theta = 1$ and $\alpha_1 = 1$ but all coefficients a_i are assumed to be positive.

same system at any positive temperature. Recently Park has shown that the Hawking temperature of exotic black holes and the black holes in the three-dimensional higher curvature gravities can be negative[35].In our opinion, possible negative temperature of black holes in their final stage of evaporation is a signature of anti-gravitation or may reflect the fact that current extensive thermodynamics is not sufficient to describe this extreme situation. Non-extensive thermodynamics of Tsallis[36] may provide a better framework for this extra-ordinary situation.

4 Summary and Conclusion

The UV/IR mixing is an outcome of spacetime noncommutativity. In a noncommutative quantum field theory, the high energy sector of the theory does not decouple from the low energy sector. This is the main modification of standard field theory in noncommutative spaces. This UV/IR mixing can be addressed in modification of standard dispersion relation. The very important outcome of these modified dispersion relations is the obvious Lorentz invariance violation at high energies. We have tried to incorporate these important notions to the issue of black hole evaporation. In the absence of supersymmetry, we have found a modified dispersion relation which has a novel energy dependence and highlights IR sector of the theory. Then using the standard argument of Bekenstein, we

have calculated entropy and temperature of TeV black holes using our new MDR. We have considered two extreme limits and in each case entropy and temperature of black hole are calculated as a function of event horizon radius. In the course of calculation, the main problem was the choice of noncommutative radius of event horizon. We have solved this problem by replacing position Dirac delta function by a smeared Gaussian distribution as has been pointed in [25]. The overall behavior of our numerical solutions are the same of existing results but there are some important differences. There is no logarithmic correction term in our entropy-area relation. Also, we find a remnant with larger mass relative to existing results.

The temperature behavior shows that noncommutativity plays the same role in general relativity as in quantum field theory, i. e., removes short distance divergences. Note also that Hawking radiation back-reaction should be considered to explain reduction of temperature in final stage of evaporation. In commutative case one expects relevant back-reaction effects during the terminal stage of evaporation because of huge increase of temperature. In our noncommutative case, the role of noncommutativity is to cool down the black hole in final stage. As a consequence, there is a suppression of quantum back reaction since the black hole emits less and less energy. Eventually, back-reaction my be important during the maximum temperature phase. Note also that, as a common belief in existing literature, the final point of evaporation is a zero-temperature TeV remnants as figure 2 shows. This is a direct consequence of minimal length due to additional gravitational uncertainty[21] and also noncommutative geometry which gives a granular structure to the spacetime manifold[38,39].

In some special circumstances our relation for temperature gives a negative temperature for a period of final stage of black hole evaporation. This negative temperature can be explained as follows: generally, negative absolute temperatures can be achieved by crossing very high temperatures. It may be a signature of anti-gravitation(repulsive behavior) or white-hole as recently has been pointed by Castro[37]. In this framework, negative temperatures (but positive entropy) are inherently associated with the repulsive gravity white-hole picture. Castro has argued that these extra-ordinary problem of having negative temperatures can be resolved by shifting of horizon location. Physically, these negative temperature may be inherent in the failure of standard thermodynamics in this extreme situation[34]. Non-extensive thermodynamics may provide a better framework for this situation. Therefore, although the general behaviors of our solutions are the same as existing picture, they can motivate some new issues in the spirit of black hole

thermodynamics.

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